

Hybrid Learned and Model-Based Regularization via Tight Framelets for Parallel MRI Reconstruction

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Abstract—Regularization plays a central role in accelerated magnetic resonance imaging (MRI) reconstruction, yet conventional approaches typically apply a uniform regularization mechanism to image components with distinct reconstruction characteristics. Model-based priors provide explicit and training-independent constraints, whereas learned priors offer strong data-driven representation capability but may exhibit reduced robustness under distribution shifts. To exploit their complementary properties, we propose Hybrid Regularization (HyReg), a learned and model-based reconstruction framework based on tight-framelet decomposition. HyReg separates the reconstruction into low-pass (LP) and high-pass (HP) components and assigns different priors to them: a learned denoising prior is applied to the LP component, while the HP components are constrained by explicit sparsity-based regularization. Moreover, a sampling-consistent LP fidelity constraint is derived from the undersampled measurement model and incorporated into a unified optimization framework. Extensive ablation and reconstruction experiments across in-distribution and out-of-distribution datasets, sampling patterns, and acceleration factors demonstrate that HyReg consistently improves reconstruction accuracy and exhibits enhanced robustness compared with representative model-based and learning-based methods.

Index Terms—Parallel MRI, image reconstruction, tight framelets, hybrid regularization, plug-and-play.

I. INTRODUCTION

Parallel MRI (pMRI) exploits complementary spatial encoding from multiple receiver coils to accelerate acquisition, but aggressive undersampling makes the reconstruction increasingly ill-posed [1]. Consequently, accurate reconstruction critically depends on how prior information is incorporated into the imaging model.

A. Existing Regularization Strategy

Classical MRI reconstruction methods incorporate explicit image priors through optimization-based regularization, including total variation (TV) [2], wavelet sparsity [3], and tight-framelet regularization [4], typically combined with data-consistency models such as SENSE [5], GRAPPA [6], and SPIRiT [7]. Owing to their interpretability, stability, and reduced reliance on large supervised datasets, these methods remain attractive for practical MRI reconstruction.

Deep learning has substantially advanced MRI reconstruction by learning powerful image priors from large-scale

datasets [8]. Model-driven frameworks that integrate measurement fidelity with learned priors have attracted particular attention [9], including plug-and-play (PnP) [10], [11], regularization-by-denoising (RED) [12]. Framelet-based LF/HF priors have also been integrated into a MAP-based deep unrolling framework [13]. Despite their strong empirical performance, learned priors may exhibit reduced robustness under distribution shifts or out-of-distribution (OOD) settings [14] and may introduce over-smoothing that attenuates fine anatomical details [15], [16]. These limitations motivate the use of complementary priors for image components with different reconstruction characteristics. HyReg therefore assigns a learned prior to the LP component while retaining explicit model-based regularization for the HF components through tight-framelet decomposition.

Despite their different formulations, existing reconstruction methods do not always account for the distinct characteristics of different image components. In the tight-framelet representation, LP components mainly characterize smooth and global image structures, while HP components encode edges and fine anatomical details that are particularly sensitive to over-smoothing. These distinct characteristics suggest that different image components may benefit from complementary forms of regularization. This motivates the central question addressed in this work: rather than designing increasingly stronger priors, can reconstruction quality be improved by assigning complementary forms of regularization to different image components?

B. Motivation for Hybrid Regularization

In our framework, tight-framelet decomposition separates the image into LP and HP components with distinct representation characteristics (Fig. 1). The LP component mainly captures smooth anatomical structures, global contrast, and slowly varying image content, whereas the HP components are dominated by sparse edges and fine anatomical details. These differences motivate component-specific regularization rather than applying a single prior uniformly to the entire image. Accordingly, HyReg applies a learned denoising prior to the LP component to suppress structured artifacts and recover global image content, while explicit sparse regularization is imposed on the HP components to provide structured, training-independent constraints. By restricting the learned prior to the

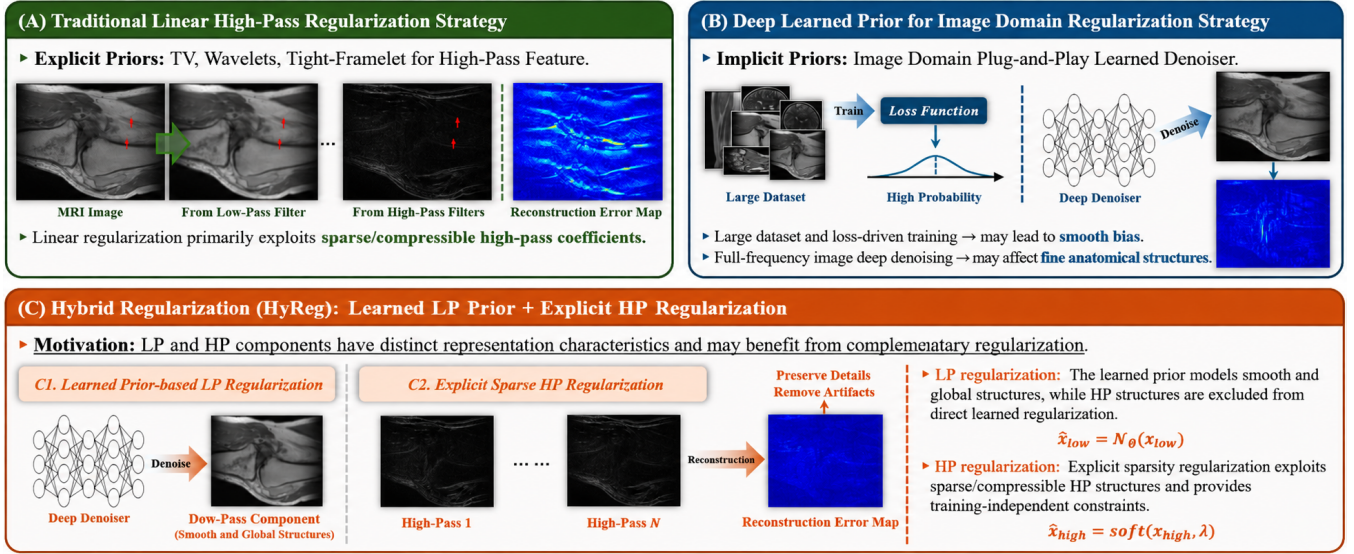


Fig. 1. Comparison of regularization strategies. (A) Linear regularization primarily exploits sparse/compressible HP coefficients. (B) Learned-prior methods regularize the entire image. (C) HyReg applies a learned prior to the LP component and explicit sparse regularization to the HP components.

LP component, HyReg also reduces its direct influence on high-frequency structures.

C. Technical Outline and Contributions

We instantiate HyReg with the SPIRiT forward model, an SGM-based learned prior for the LP component [17], DHF-based sparse regularization for the HP components [4], and an LP-specific fidelity constraint derived from the undersampled measurements. Ablation studies evaluate the contributions of these components, with robustness further assessed under aggressive undersampling and noise. The resulting framework integrates learned LP regularization with explicit model-based HP regularization within a unified optimization model, reducing direct reliance on learned priors for HP reconstruction.

The main contributions are summarized as follows.

- 1) **Hybrid regularization principle.** We propose a HyReg framework for pMRI that assigns complementary roles to learned and model-based priors, applying learned regularization to the LP component while retaining explicit sparse regularization for the HP components.
- 2) **Low-pass data consistency.** We derive an LP-specific fidelity constraint from the tight-framelet decomposition and undersampled measurements, providing explicit measurement consistency for the learned LP reconstruction.
- 3) **Coupled reconstruction model.** We develop a unified optimization model integrating the learned LP prior, LP-specific fidelity, and explicit HP regularization, together with an efficient alternating optimization algorithm compatible with standard model-based pipelines.

II. PRELIMINARIES

A. MRI Sampling and k -space Forward Model

Let $x \in \mathbb{C}^n$ denote the fully sampled multi-coil image to be reconstructed, and $\mathcal{P} \in \{0, 1\}^{n \times n}$ denote the undersampled mask. The undersampled measurement model is given by

$$y = \mathcal{P}\mathcal{F}x + \eta, \quad (1)$$

where y is the acquired k -space data, $\mathcal{F}$ is the Fourier transform, and η denotes measurement noise.

Following SPIRiT [7] and COMEUS [18], a calibration operator $\mathcal{G}$ is estimated from the ACS region to enforce k -space self-consistency, i.e., $\mathcal{F}x \approx \mathcal{G}(\mathcal{F}x)$. With $\mathcal{Q} = \mathcal{I} - \mathcal{P}$, substituting the data-consistent k -space $\mathcal{Q}\mathcal{F}x + y$ into this relation yields

$$\mathcal{A}x \approx b, \quad (2)$$

where $\mathcal{A} = (\mathcal{G} - \mathcal{I})\mathcal{Q}\mathcal{F}$ and $b = (\mathcal{I} - \mathcal{G})y$.

B. Deep Denoiser-Based Plug-and-Play Regularization

Based on the calibration model in (2), a learned prior can be incorporated into the reconstruction through the following regularized formulation:

$$\min_x \frac{1}{2} \|\mathcal{A}x - b\|_2^2 + \mathcal{R}_\Theta(x), \quad (3)$$

where $\mathcal{R}_\Theta(\cdot)$ denotes an implicit learned regularizer parameterized by Θ . Following the plug-and-play (PnP) framework [10], the proximal operation associated with $\mathcal{R}_\Theta$ is replaced by a learned denoiser $\mathcal{N}_\Theta(\cdot)$. In this work, the resulting problem is solved using ADMM-PnP, where the data-fidelity subproblem is handled by conjugate gradient (CG) and the regularization subproblem is implemented through $\mathcal{N}_\Theta$. This formulation serves as the image-domain learned-prior baseline and forms the basis for the LP regularization introduced in Sec. III.

III. LOW-PASS FIDELITY AND HYBRID REGULARIZATION

A. LP Fidelity from Sampling Consistency

We first derive a sampling-consistent observation for the LP component. For notational simplicity, measurement noise is omitted in the following derivation. Let $x_\omega = \omega * x$, where ω is a circular convolution filter. By the convolution theorem, $\hat{x}_\omega = \hat{\omega} \odot \hat{x}$. Since the sampling operator $\mathcal{P}$ acts pointwise in k -space, it commutes with pointwise multiplication, and (1) gives

$$\mathcal{P}\hat{x}_\omega = \mathcal{P}(\hat{\omega} \odot \hat{x}) = \hat{\omega} \odot (\mathcal{P}\hat{x}) = \hat{\omega} \odot y.$$

For the LP filter ω_0 , define $x_\ell = \omega_0 * x$ and $y_\ell = \hat{\omega}_0 \odot y$. Then $\mathcal{P}\mathcal{F}x_\ell = y_\ell$, providing a sampling-consistent observation of the LP component. Following the SPIRiT formulation, an LP calibration operator $\mathcal{G}_\ell$ estimated from y_ℓ yields

$$\mathcal{A}_\ell x_\ell \approx b_\ell, \quad \mathcal{A}_\ell = (\mathcal{G}_\ell - \mathcal{I})\mathcal{Q}\mathcal{F}, \quad b_\ell = (\mathcal{I} - \mathcal{G}_\ell)y_\ell. \quad (4)$$

This construction provides an explicit measurement-consistency constraint for the learned LP reconstruction through the LP-specific self-consistency operator $\mathcal{G}_\ell$.

B. Explicit High-Pass Sparse Regularization

With the DHF system [4], we define the analysis operator $\mathcal{W} = [\mathcal{W}_\ell^\top, \mathcal{W}_h^\top]^\top$, where $\mathcal{W}_\ell$ and $\mathcal{W}_h$ denote the LP and HP components, respectively. The HP components are constrained using the following sparsity-promoting explicit regularizer:

$$\mathcal{R}_h(x) = \|\Lambda \mathcal{W}_h x\|_1, \quad (5)$$

where Λ is a diagonal weighting matrix following [18].

C. Unified Hybrid Reconstruction Model

Based on the LP fidelity constraint, we formulate the proposed HyReg model as

$$\begin{aligned} \min_{x, x_\ell} & \frac{1}{2} \|\mathcal{A}_\ell x_\ell - b_\ell\|_2^2 + \mathcal{R}_\Theta(x_\ell) + \frac{1}{2} \|\mathcal{A}x - b\|_2^2 + \mathcal{R}_h(x) \\ \text{s.t.} & \quad x_\ell = \mathcal{W}_\ell x. \end{aligned}$$

The implicit regularizer $\mathcal{R}_\Theta(\cdot)$ is dedicated to LP reconstruction, while the HP components retain explicit sparse regularization, thereby integrating learned and model-based priors across the tight-framelet components.

We solve HyReg using half-quadratic splitting (HQS) [19]. Introducing an auxiliary variable $x_\ell = \mathcal{W}_\ell x$, the original model is decoupled into the following alternating subproblems:

$$\begin{aligned} x_\ell^{t+1} &= \arg \min_{x_\ell} \frac{1}{2} \|\mathcal{A}_\ell x_\ell - b_\ell\|_2^2 + \mathcal{R}_\Theta(x_\ell) + \frac{\mu}{2} \|x_\ell - \mathcal{W}_\ell x^t\|_2^2, \\ x^{t+1} &= \arg \min_x \frac{1}{2} \|\mathcal{A}x - b\|_2^2 + \mathcal{R}_h(x) + \frac{\mu}{2} \|\mathcal{W}_\ell x - x_\ell^{t+1}\|_2^2. \end{aligned}$$

Here, $\mu > 0$ denotes the penalty parameter, which is fixed to $\mu = 1$ in all experiments. Defining

$$\mathcal{B}_\ell = \begin{bmatrix} \mathcal{A}_\ell \\ \mathcal{I} \end{bmatrix}, g_\ell = \begin{bmatrix} b_\ell \\ \mathcal{W}_\ell x^t \end{bmatrix}, \mathcal{B} = \begin{bmatrix} \mathcal{A} \\ \mathcal{W}_\ell \end{bmatrix}, g = \begin{bmatrix} b \\ x_\ell^{t+1} \end{bmatrix}.$$

Then the above problem can be rewritten as

$$x_\ell^{t+1} = \arg \min_{x_\ell} \frac{1}{2} \|\mathcal{B}_\ell x_\ell - g_\ell\|_2^2 + \mathcal{R}_\Theta(x_\ell), \quad (6a)$$

$$x^{t+1} = \arg \min_x \frac{1}{2} \|\mathcal{B}x - g\|_2^2 + \mathcal{R}_h(x). \quad (6b)$$

The LP subproblem (6a) is solved using the ADMM-PnP scheme described in Sec. II-B, while the image reconstruction subproblem (6b), equipped with explicit HP sparse regularization is solved using PD3O [20]. Detailed iterative updates and convergence analysis follow [18], [21].

Compared with applying multiple regularizers uniformly to the reconstructed image, the proposed formulation reflects the central principle of this work: complementary forms of regularization are assigned to image components with distinct representation characteristics. In particular, the learned prior is applied to the LP component, while the HP components are constrained by explicit, training-independent regularization.

IV. ABLATION STUDY AND ANALYSIS

A. Experimental Setup

We ablate the three components of HyReg (6): the LP fidelity $\mathcal{D}_\ell(\cdot) = \frac{1}{2} \|\mathcal{A}_\ell \cdot - b_\ell\|_2^2$, LP learned regularization $\mathcal{R}_\Theta$, and HP sparse regularization $\mathcal{R}_h$. Removing all three reduces HyReg to the original SPIRiT model. The resulting variants are summarized in Table I.

Implementation. We adopt the SGM-based denoiser of [22], trained on fastMRI¹. For PD3O, we set $\rho = 1.99/L$ and $\gamma = 0.99/\rho$, where L is the Lipschitz constant of the quadratic term in (6b). All methods are run for a maximum of 80 iterations.

Datasets and Sampling. Experiments are conducted on the OOD 3T MRI dataset [23] and NYU dataset² using uniform Cartesian undersampling with varying acceleration factors R and numbers of ACS lines.

Metrics. We evaluate reconstruction quality using PSNR, SSIM, HFEN, and HaarPSI, where HFEN and HaarPSI characterize high-frequency and perceptual structural fidelity, respectively [24], [25].

Baselines. We compare HyReg with the image-domain PnP model (3) using the same SGM denoiser, and the two-level tight-framelet (TNTF) model [26], which further decomposes the LP component of the single-level DHF.

B. Quantitative Experiment

Numerical experiments are performed on the 3T MRI brain dataset with image size 512×512 and 12 coils. Cartesian uniform undersampling with 24 ACS lines is used, and the acceleration factor ranges from $R = 2$ to $R = 10$. For each R , 10 independent experiments are conducted.

Table I reports results averaged over all R , while Fig. 2 shows performance across different R . The ablations demonstrate the contributions of LP fidelity, LP learned regularization, and HP sparse regularization. Although PnP becomes

¹<http://fastmri.med.nyu.edu/>

²<http://mridata.org/>

TABLE I

ABLATION STUDY OF DIFFERENT REGULARIZATION STRATEGIES (MEAN $\pm$ STANDARD DEVIATION). SHADING AND UNDERLINING INDICATE THE BEST MEAN AND SMALLEST DEVIATION, RESPECTIVELY.

Method	PSNR $\uparrow$	SSIM $\uparrow$	HFEN $\downarrow$	HaarPSI $\uparrow$
PnP-SGM	30.526 $\pm$ 1.578	0.835 $\pm$ 0.024	0.435 $\pm$ 0.077	0.769 $\pm$ 0.026
TNTF	32.533 $\pm$ 1.914	0.880 $\pm$ 0.017	0.346 $\pm$ 0.059	0.832 $\pm$ 0.025
HyReg	34.247 $\pm$ 1.872	0.909 $\pm$ 0.014	0.297 $\pm$ 0.053	0.869 $\pm$ 0.023
w/o $\mathcal{D}_\ell$	33.668 $\pm$ 2.016	0.905 $\pm$ 0.015	0.311 $\pm$ 0.053	0.853 $\pm$ 0.024
w/o $\mathcal{R}_\Theta$	31.339 $\pm$ 1.619	0.853 $\pm$ 0.016	0.412 $\pm$ 0.059	0.801 $\pm$ 0.020
w/o $\mathcal{R}_h$	33.943 $\pm$ 1.808	0.904 $\pm$ 0.014	0.303 $\pm$ 0.051	0.866 $\pm$ 0.022
w/o $\mathcal{D}_\ell$ & $\mathcal{R}_\Theta$	31.537 $\pm$ 1.803	0.869 $\pm$ 0.016	0.385 $\pm$ 0.057	0.805 $\pm$ 0.021
SPIRiT only	30.267 $\pm$ 1.638	0.840 $\pm$ 0.017	0.424 $\pm$ 0.053	0.783 $\pm$ 0.019

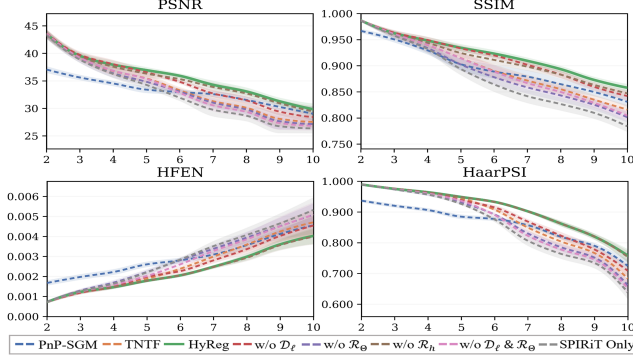


Fig. 2. Reconstruction performance under different acceleration factor R . The curves show the averaged results and the shaded regions indicate the standard deviation. The proposed HyReg framework consistently achieves the best performance, especially at higher acceleration factors.

more competitive at higher R , HyReg consistently achieves better performance, supporting the complementary roles of learned LP and sparse HP priors. Moreover, HyReg outperforms TNTF despite its multi-level linear representation, indicating that the improvement cannot be attributed solely to increased decomposition depth.

C. Frequency-Domain Analysis

To examine the frequency-dependent behavior of different regularization strategies, we compute the log spectral error $E(\kappa) = \log(1 + |\mathcal{F}(u - u^*)(\kappa)|)$, where u and u^* denote the reconstruction and reference, respectively. Radial averaging yields a one-dimensional profile over the normalized frequency radius, which is divided into low (0–0.10), mid (0.10–0.40), and high (0.40–1.00) bands.

As shown in Fig. 3, the proposed HyReg achieves the lowest low- and mid-frequency errors while remaining competitive in the high-frequency band, supporting the complementary roles of learned LP and sparse HP regularization.

D. Visual Comparison

Fig. 4 presents the visual ablation results under aggressive undersampling ($R = 6$, $ACS = 22$) on an OOD NYU knee slice (image size 768×616 with 15 coils). The red arrows indicate that only (b), (d), and (g) effectively remove undersampling-induced aliasing artifacts. Nevertheless, the

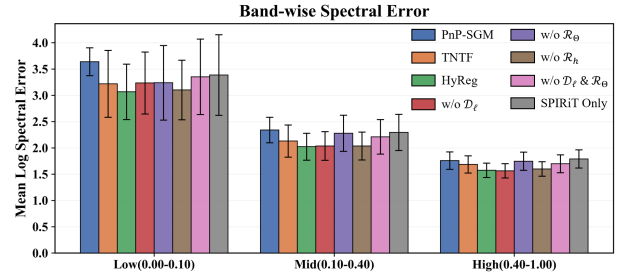


Fig. 3. Mean log spectral error over low-, mid-, and high-frequency bands.

yellow arrows reveal that the PnP-SGM method in (b) over-smooths anatomical structures and loses fine details. HyReg suppresses aliasing while retaining structural details and yields the lowest reconstruction error.

V. EXPERIMENTS AND RESULTS

A. Experimental Setup

Unless otherwise specified, all settings follow Sec. IV-A. We compare HyReg with ESPIRiT [27], MoDL [28], FRSGM [22], and HFS-SDE [9]. HFS-SDE also employs frequency separation but follows a different reconstruction strategy. MoDL, FRSGM, and HFS-SDE are trained on fastMRI, while HyReg uses the same SGM denoiser as FRSGM. For HFS-SDE, we use the official implementation and the pre-trained model released by the authors without retraining.

FastMRI is used for in-distribution (ID) evaluation, while OOD evaluation uses the NYU MRI and 3T brain and phantom datasets [23].

B. Efficiency and Memory Evaluation

Table II compares runtime, peak GPU memory consumption, and model size on 320×320 16-coil fastMRI brain data using a Tesla P100 GPU. The 320×320 resolution is used to accommodate HFS-SDE.

TABLE II

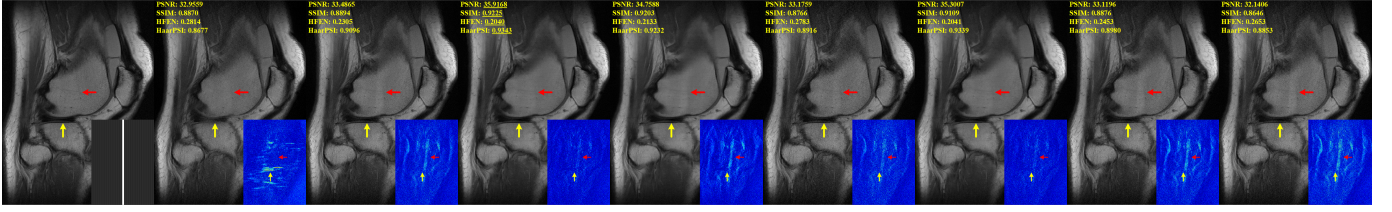
COMPARISON OF COMPUTATIONAL COST, INCLUDING RUNTIME (S), PEAK MEMORY CONSUMPTION (MB), AND MODEL SIZE (M PARAMETERS), FOR MRI RECONSTRUCTION WITH IMAGE SIZE 320×320 AND 16 COILS.

Methods	ESPIRiT	MoDL	FRSGM	HFS-SDE	HyReg
Runtime	22.95	173.26	55.60	359.53	102.09
Memory	270.20	265.23	1265.32	1111.71	2504.02
#Params	-	0.69	34.43	119.36	8.25

HyReg requires higher GPU memory due to its multi-coil denoising and frequency decomposition, but uses only 8.25M parameters and has lower runtime than MoDL and HFS-SDE.

C. Numerical and Statistical Evaluation

We evaluate reconstruction performance on ID and OOD datasets under Cartesian uniform undersampling with 24 ACS lines and acceleration factors ranging from $R = 2$ to $R = 10$. For each setting, at least 90 independent runs are performed. The quantitative results are summarized in Table III. HyReg



(a) Target&Mask (b) PnP-SGM (c) TNTF (d) HyReg (e) w/o $\mathcal{D}_\ell$ (f) w/o $\mathcal{R}_\Theta$ (g) w/o $\mathcal{R}_h$ (h) w/o $\mathcal{D}_\ell \& \mathcal{R}_\Theta$ (i) SPIRiT Only

Fig. 4. Visual comparison under aggressive undersampling ($R = 6$, $ACS = 22$). Red and yellow arrows indicate residual artifacts and fine-detail loss, respectively. Quantitative metrics are reported in the upper-left corner of each image, with the best values underlined. Error maps are displayed in the bottom-left corner.

consistently achieves the best or near-best performance across all evaluated metrics on both ID and OOD datasets. Notably, the performance gains are more pronounced on OOD datasets, indicating that the proposed HyReg strategy provides improved robustness and generalization under domain shifts.

TABLE III

ID AND OOD RECONSTRUCTION RESULTS (MEAN $\pm$ STANDARD DEVIATION). **SHADING** AND **UNDERLINING** INDICATE THE BEST MEAN AND SMALLEST DEVIATION, RESPECTIVELY.

Data	Method	PSNR $\uparrow$	SSIM $\uparrow$	HFEN $\downarrow$	HaarPSI $\uparrow$
ID	ESPIRiT	26.609 $\pm$ 2.396	0.473 $\pm$ 0.101	0.598 $\pm$ 0.097	0.694 $\pm$ 0.082
	MoDL	26.815 $\pm$ 2.490	0.487 $\pm$ 0.104	0.593 $\pm$ 0.095	0.701 $\pm$ 0.081
	FRSGM	29.658 $\pm$ 2.372	0.569 $\pm$ 0.077	<u>0.329$\pm$0.056</u>	0.833 $\pm$ 0.061
	HFS-SDE	28.581 $\pm$ 2.121	0.546 $\pm$ 0.079	0.444 $\pm$ 0.067	0.775 $\pm$ 0.059
	HyReg	<u>32.345$\pm$1.465</u>	<u>0.830$\pm$0.055</u>	0.351 $\pm$ 0.057	<u>0.867$\pm$0.029</u>
OOD	ESPIRiT	21.625 $\pm$ 5.895	0.475 $\pm$ 0.135	0.980 $\pm$ 0.524	0.450 $\pm$ 0.181
	MoDL	21.881 $\pm$ 6.019	0.493 $\pm$ 0.137	0.952 $\pm$ 0.514	0.460 $\pm$ 0.181
	FRSGM	23.849 $\pm$ 5.908	0.625 $\pm$ 0.196	0.746 $\pm$ 0.398	0.530 $\pm$ 0.204
	HFS-SDE	23.596 $\pm$ 5.909	0.566 $\pm$ 0.182	0.830 $\pm$ 0.565	0.519 $\pm$ 0.194
	HyReg	<u>32.376$\pm$2.338</u>	<u>0.839$\pm$0.052</u>	<u>0.390$\pm$0.223</u>	<u>0.828$\pm$0.059</u>

D. Visual Comparisons

We further compare visual quality on OOD datasets. Due to the resolution constraint of HFS-SDE, all images are centrally cropped to 320×320 . Fig. 6 shows reconstruction results on brain (top row, 512×512 , 12 coils, Cartesian random sampling) and phantom (bottom row, 512×512 , 4 coils, non-Cartesian spiral sampling) data.

For the brain case, ESPIRiT and MoDL exhibit noticeable residual noise that degrades fine anatomical details (yellow arrows), while FRSGM and HFS-SDE suffer from residual artifacts (red arrows). For the phantom case, ESPIRiT, MoDL, and HFS-SDE show varying degrees of aliasing artifacts (red arrows). In particular, HFS-SDE, which performs k -space low-/high-frequency separation learning, shows limited generalization on the structurally distinct phantom data. Although FRSGM effectively suppresses artifacts, it tends to over-smooth the reconstruction, resulting in blurred details due to domain shift (yellow arrows).

Overall, the proposed HyReg effectively mitigates these issues and achieves the lowest reconstruction error and the best quantitative performance.

VI. CONCLUSION

We propose HyReg, a hybrid learned and model-based regularization framework for parallel MRI reconstruction. HyReg decomposes reconstruction into complementary LP and HP subproblems via tight-framelet decomposition: the LP component is processed by a deep denoiser, while the HP components are constrained by tight-framelet sparsity. This component-specific design enables a principled integration of learned priors and model-based regularization within a unified optimization framework. Extensive experiments demonstrate improved reconstruction quality and robustness under both ID and OOD settings, and ablation studies further validate each component.

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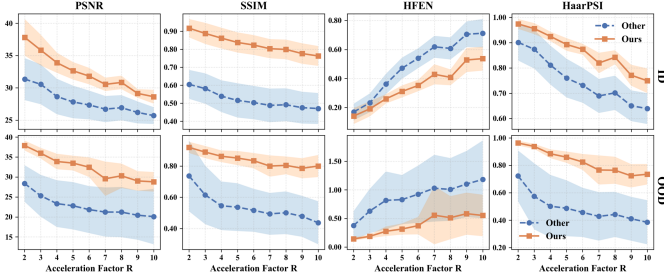


Fig. 5. Performance comparison of the proposed HyReg and competing methods under different acceleration factors R ranging from 2 to 10. Top and bottom rows correspond to the ID and OOD results, respectively.

Fig. 5 illustrates the performance trends of HyReg and competing methods under different acceleration factors R based on the experiments reported in Table III. HyReg consistently maintains superior performance across the entire range of $R = 2$ to 10, while exhibiting relatively stable degradation as the undersampling rate increases. These results demonstrate the robustness of HyReg to varying sampling conditions.

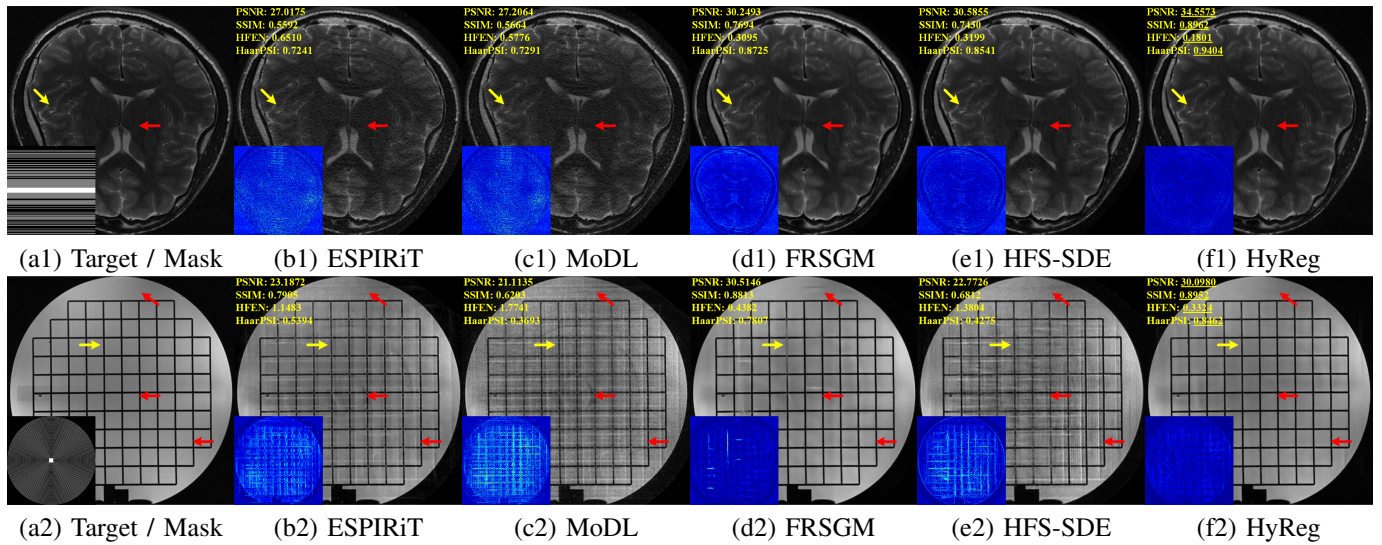


Fig. 6. Visual comparison on 3T MRI brain (top row) and phantom (bottom row) data under (a1) Cartesian random sampling (ACS = 19, sampling rate = 31%) and (a2) non-Cartesian spiral sampling (ACS = 24, sampling rate = 16%). Red and yellow arrows indicate residual artifacts and fine-detail loss, respectively. Metrics are shown in the upper-left corner, with best values underlined. Error maps are shown in the bottom-left corner.

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